Probability of Implication

ZHAN, Likan
zhanlikan@blcu.edu.cn
https://likan.org

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Beijing Language and Culture University

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Psychology as Science for Human Mind

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- Physics

  Appropriate model for the external physical world.

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- Psychology

  Appropriate framework for the internal cognitive world.

Truth-Conditional Semantics

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- Knowing the meaning of a sentence is knowing under which circumstances it is true or false. (Davidson, 1967)

Principle of Compositionality

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- A truth-functional compound proposition is a proposition whose truth or falsity (that is, truth-value) is a function of the truth or falsity of its component propositions. (Mosley & Baltazar, 2019)

Sentential Connectives and Logical Operators

- Apparent parallel between human language and Boolean logic

Name	Language	Boolean logic
Negation	*not*	$\neg$
Conjunction	*and*	$\wedge$
Disjuntion	*or*	$\vee$
Conditional	*If⋯then*	$\supset$

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- Denote *If A then* C as $A > C$

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Material Implication in Boolean Logic

A	C	$A \supset C$
False	False	True
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True	False	False
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Paradox of Material Implication

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- $A \supset C = \neg A \vee C$

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- $A \supset C = \neg A \lor C$
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Probabilites of Material Implication

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- Probabilities of Material Implication

$$\begin{aligned} Pr(A > C) &= Pr(A \supset C) \\ &= Pr(A \wedge C) + Pr(\neg A \wedge C) + Pr(\neg A \wedge \neg C) \\ &= 1 - Pr(A \wedge \neg C) \end{aligned}$$

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- The sum of thre three probabilities **is not** the significant predictor of the judged subjective probability of A > C. (Evans et al., 2003; Oberauer & Wilhelm, 2003; Over et al., 2007; Singmann et al., 2014)

Probabilities of Conditional Statements

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- Condiitonal Probability

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- Conditional Probability $Pr(C|A)$ **is** the significant predictor of the judged subjective probability of A > C. (Evans et al., 2003; Fugard et al., 2011; Girotto & Johnson-Laird, 2004; Oberauer & Wilhelm, 2003; Oberauer et al., 2007; Over et al., 2007; Singmann et al., 2014; Skovgaard-Olsen et al., 2016, 2019)

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  $A \wedge C \Rightarrow A \supset C$
- $A > C \equiv A \supset C = \neg A \vee C$
  $A \wedge C \Rightarrow A > C$
- *If Napoleon is dead, Oxford is in England.*

Default and Penalty Hypothesis

$$\Delta p_1 = [Pr(A \wedge C) + Pr(\neg A \wedge \neg C)] - [Pr(\neg A \wedge C) + Pr(A \wedge \neg C)]$$

$$\Delta p_2 = \frac{Pr(C|A) - Pr(C)}{1 - Pr(C)}$$

$$\Delta p_3 = Pr(C|A) - Pr(C|\neg A)$$

$$\Delta p_4 = \frac{Pr(C|A) - Pr(C|\neg A)}{1 - Pr(C|\neg A)} = \frac{Pr(C|A) - Pr(C)}{Pr(\neg A \wedge \neg C)}$$

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- Positive evidence
  (Krzy anowska et al., 2017; Skovgaard-Olsen et al., 2016, 2019)
- Negative evidence
  (Oberauer et al., 2007; Over et al., 2007; Singmann et al., 2014)
- Our results suggest that the positive results are confounded by other factors. (Zhan & Wang, In Preparation)

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Embeddings of conditionals

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- Negated conditionals: $\neg(A > C)$

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- Right-nested conditionals: $A > (B > C)$

Stalnaker's Hypothesis and Factorization Hypothesis

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- Stalnaker's Hypothesis (Stalnaker, 1970): For every probability function Pr and for every conditional $A > C$, possibly complex:

$$Pr(A > C) = Pr(C|A),$$

provided that $Pr(A) > 0$.

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- Factorization Hypothesis (Fitelson, 2015): For every probability function Pr and for all sentences A and B such that $Pr(A \wedge B) > 0$,

$$Pr(B > C|A) = Pr(C|A \wedge B)$$

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- Factorization Hypothesis (Fitelson, 2015): For every probability function Pr and for all sentences A and B such that $Pr(A \wedge B) > 0$,

$$Pr(B > C|A) = Pr(C|A \wedge B)$$

- Import-Export Principle: $A \supset (B \supset C) \equiv (A \wedge B) \supset C$

$$Pr(A > (B > C)) = Pr(B > C|A) = Pr(C|A \wedge B)$$

Right-Nested Conditionals and Triviality Theorem

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- Triviality Theorem (Lewis, 1976): If A is probabilistically compatible with both C and $\neg C$, that is, if $Pr(A \wedge C) > 0$ and $Pr(A \wedge \neg C) > 0$, then $Pr(A > C) = Pr(C)$.

Right-Nested Conditionals and Triviality Theorem

- Triviality Theorem (Lewis, 1976): If A is probabilistically compatible with both C and $\neg C$, that is, if $Pr(A \wedge C) > 0$ and $Pr(A \wedge \neg C) > 0$, then $Pr(A > C) = Pr(C)$.
- Proof

$$
\begin{aligned}
Pr(A > C|C) &= Pr(C|A \wedge C) = 1 \\
Pr(A > C|\neg C) &= Pr(C|A \wedge \neg C) = 0 \\
Pr(A > C) &= Pr(A > C|C)Pr(C) + Pr(A > C|\neg C)Pr(\neg C) \\
&= 1 \cdot Pr(C) + 0 \cdot Pr(\neg C) \\
&= Pr(C)
\end{aligned}
$$

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Hypothetical Properties of Conditionals

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- *The apple is green* versus *If the apple is green*.

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- A, C, $A \wedge C$ versus $A > C$. (Zhan et al., 2015, 2018; Zhan & Zhou, 2023)

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- A, C, $A \wedge C$ versus $A > C$. (Zhan et al., 2015, 2018; Zhan & Zhou, 2023)
- A, C, $A \wedge C$ versus $A \vee C$. (Zhan, 2018)

Go Back to the Paradoxes

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- *If the moon is made of green cheese, then life exists on other planets.*

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- *If the moon is made of green cheese, then life exists on other planets.*
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- *If the moon is made of green cheese, then life exists on other planets.*
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Conditionals as Operators or Gates

- The effect of conditional $A > C$ happens before measurement which does not make the superposition of states to collapse.

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- The effect of conditional $A > C$ happens before measurement which does not make the superposition of states to collapse.
- The conditional $A > C$ should be regarded as an intact unit.

Conditionals as Controled-NOT Gate?

Conditionals as Controled-NOT Gate?

- Material Implication

A	C	$A \supset C$
False	False	True
False	True	True
True	False	False
True	True	True

Conditionals as Controled-NOT Gate?

- Material Implication

A	C	$A \supset C$
False	False	True
False	True	True
True	False	False
True	True	True

- Controled-NOT Gate

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Thank you for your attention !

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